A Time-domain Approach of Ion Flow Field around AC-DC hybrid Transmission Lines Based on Method of Characteristics

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When there is a HVAC power transmission line near a HVDC transmission line, the nonlinear ion flow field around the DC transmission line due to corona becomes more complex because the AC field affects the movement of the ions in the space. In this paper, a pair of first-order partial differential equations of the ion densities in time domain is derived. Then, a time-domain iterative approach based on the method of characteristics is presented to calculate the ion flow field around AC-DC hybrid transmission line. The advantage of the method is that almost all the variables can be obtained via analytic formulas and no large-scale system of equations needs to be solved. The calculated and the measured results are in good agreement.

***Index Terms*—Corona, electromagnetic fields, transmission lines, time-domain analysis.**

I. Introduction

Due to the lack of rights-of-way (ROW), DC and AC power transmission lines sharing the same ROW has become a trend. In this situation, the nonlinear ion flow fields around HVDC transmission lines due to corona become more complex. Firstly, the surface field of the DC conductors is modified by a superposed AC component. The intensity of the corona discharge on the DC conductors will vary with the AC field. Secondly, the trajectory of the space charges is modified by the AC field [1]. As the space charges can affect the original field significantly, it is interesting to analyze the ion flow field around AC-DC hybrid transmission lines.

Many methods, such as FEMs, finite volume method, meshless method, and some integral equation methods have already been used [2-5] to calculate the stable ion flow field around HVDC transmission lines. Most of them need to set up large-scale system of equations. As the ion flow field around AC-DC hybrid transmission lines is a time-varying field, the scale of the equations will be very large if the above methods are used [6]. Paper [7] used the method of characteristics to calculate the ion flow fields. The advantage of the method is that almost all the variables can be obtained via analytic formulas and no large-scale system of equations needs to be solved, which means the method is suitable for large-scale calculation. However, only stable field has been solved until now.

In this paper, a pair of time-domain first-order partial differential equations determining the distribution of ion densities is derived. An iterative approach based on the method of characteristics is presented to calculate the ion flow field around AC-DC hybrid transmission line in time domain.

II. Basic Equations

In time domain, the ion flow field should obey following equations [6]:

$$\nabla\cdot\boldsymbol{E}=(\rho^{-}-\rho^{+})/\varepsilon_0 \tag{1}$$

$$\begin{cases}\boldsymbol{J}^{+}=\rho^{+}(k^{+}\boldsymbol{E}+\boldsymbol{W})\\ \boldsymbol{J}^{-}=\rho^{-}(k^{-}\boldsymbol{E}-\boldsymbol{W})\end{cases} \tag{2}$$

$$\begin{cases}\nabla\cdot\boldsymbol{J}^{+}=-\dfrac{R\rho^{+}\rho^{-}}{e}-\dfrac{\partial\rho^{+}(t)}{\partial t}\\ \nabla\cdot\boldsymbol{J}^{-}=\dfrac{R\rho^{+}\rho^{-}}{e}+\dfrac{\partial\rho^{-}(t)}{\partial t}\end{cases} \tag{3}$$

where $\boldsymbol{E}$ is the electric field strength, ρ^{+} and ρ^{-} are the absolute values of positive and negative ion densities, ε_0 is the permittivity of air, $\boldsymbol{J}^{+}$ and $\boldsymbol{J}^{-}$ are positive and negative ion current density, k^{+} and k^{-} are positive and negative ion mobilities which describe the effect of the electric field on the movement of the ions in the space, $\boldsymbol{W}$ is wind velocity vector, R is the coefficient of recombination of ions, and e is electron charge (1.602×10^{-19} C). The potential on the ground plane is zero, and the potentials on the surfaces of the conductors are known.

III. Method of Characteristics

By substituting (1) and (2) into (3), a pair of first-order partial differential equations is derived:

$$\begin{cases}(k^{+}\boldsymbol{E}+\boldsymbol{W})\cdot\nabla\rho^{+}+\dfrac{\partial\rho^{+}(t)}{\partial t}=\dfrac{k^{+}{\rho^{+}}^{2}}{\varepsilon_0}-(\dfrac{k^{+}}{\varepsilon_0}+\dfrac{R}{e})\rho^{+}\rho^{-}\\ (k^{-}\boldsymbol{E}-\boldsymbol{W})\cdot\nabla\rho^{-}-\dfrac{\partial\rho^{-}(t)}{\partial t}=-\dfrac{k^{-}{\rho^{-}}^{2}}{\varepsilon_0}+(\dfrac{k^{-}}{\varepsilon_0}+\dfrac{R}{e})\rho^{+}\rho^{-}\end{cases} \tag{4}$$

The method of characteristics can convert a partial differential equation into an ordinary differential equation, and an analytic formula can be derived. For the fields near the transmission line, 2D model can be used. For example, the equation for positive ions can be derived from the first equation of (4).

Let $A_x=k^{+}E_x+W_x$, $A_y=k^{+}E_y+W_y$, $B=k^{+}/\varepsilon_0$, and $D=-(k^{+}/\varepsilon_0+R/e)\rho^{-}$, then along the "characteristic line" defined by $\mathrm{d}x/\mathrm{d}t=A_x$ and $\mathrm{d}y/\mathrm{d}t=A_y$, the first equation of (4) becomes an ordinary differential equation:

$$\begin{aligned}&(k^{+}\boldsymbol{E}+\boldsymbol{W})\cdot\nabla\rho^{+}+\frac{\partial\rho^{+}(t)}{\partial t}=\frac{\partial\rho^{+}}{\partial t}+A_x\frac{\partial\rho^{+}}{\partial x}+A_y\frac{\partial\rho^{+}}{\partial y}=\\ &\frac{\partial\rho^{+}}{\partial t}+\frac{\partial\rho^{+}}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{\partial\rho^{+}}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}\rho^{+}(t,x,y)}{\mathrm{d}t}=B{\rho^{+}}^{2}+D\rho^{+}\end{aligned} \tag{5}$$

If the characteristic lines are divided into short segments and in each segment B and D are constants, the solution of above equation in the segment along the characteristic lines would be

$$\rho^+ = \frac{1}{1-e^{Dt+C}}\frac{D}{B}e^{Dt+C}, \tag{6}$$

where C is a constant due to integral, which can be decided if ρ^+ at a point on the "characteristic line" is known. If the space is divided in to several elements and the electric field and ρ^- are constants on each element, the new position of the element (x_2, y_2) and the value of the ion density ρ_2^+ at time t_2 can be determined by the corresponding values at time t_1 along the "characteristic line":

$$\rho_2^+ = \frac{1}{1-e^{Dt_2+C}}\frac{D}{B}e^{Dt_2+C}, \tag{7}$$

$$x_2 = x_1 + (k^+E_x + W_x)(t_2 - t_1),$$

$$y_2 = y_1 + (k^+E_y + W_y)(t_2 - t_1),$$

where $C = \ln\frac{\rho_1^+}{\rho_1^+ + D/B} - Dt_1$.

IV. Ion Flow Field Calculation

Since the electric field and the ions affect each other, it is a nonlinear problem, which can be solved by an iterative method. The main iterative process of the method is:

1) Initialization

The region of interest is subdivided into small elements. The iterative process is started by assigning initial values to ρ^+ and ρ^- in the small elements. These initial values can be the stable ion flow field around the HVDC transmission lines when there are no voltages on the AC transmission lines, which can be obtained by the method mentioned in the introduction of this paper [2-5].

2) Updating ion density in the space

Let the voltages on the AC transmission lines vary with time, the electric field in the space at time $t_2 = t_1 + \Delta t$ can be calculated by accumulating all the contributions from the ions and the charges on the conductors. Thus, for each element, the value of the ion density ρ_2^+ at time t_2 and the new position (x_2, y_2) can be determined from (7). Then, a linear interpolation method is used to obtain the new ion densities in all the elements because the ions obtained from (7) have moved away from the elements.

3) Updating ion density next to the DC conductor

The electric fields on the surfaces of the DC conductors are calculated with the updated ions and the charges on the conductors at time t_2. Then, the ion densities of the element next to the DC conductors will be reset according to the gap between the calculated electric field and the onset value of corona to keep the electric field on the DC conductor surface at the threshold [3, 4].

4) Repetition Process

Step 2 and step 3 will be repeated for several power periods until the gap of the peak ion densities next to the DC conductors obtained in two consecutive periods is smaller than a specified tolerance (in this paper, it is 1%).

Because just analytic formulas are used in the approach except for the linear interpolation, the requirement for computer memory is low.

V. Validation

Both DC and AC components of the ground level electric field and ion current density under a DC conductor have been measured when an AC conductor was nearby [1]. Fig. 1 shows the assignment of the experiment. Fig.2 and Fig. 3 show the measured and calculated DC components. Due to limited space, the AC components will be presented in the final paper. The results are in good agreement.

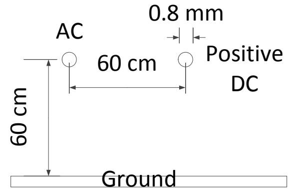

Fig.1. Reduced-scale AC-DC hybrid line model.

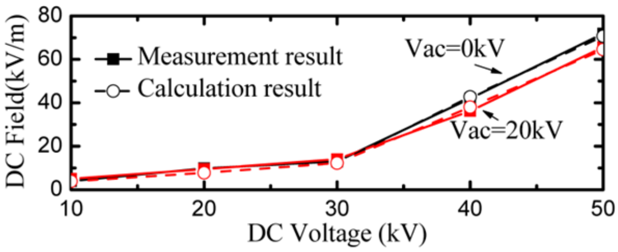

Fig.2. Ground level DC electric field

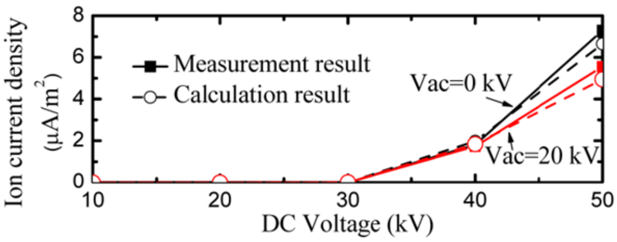

Fig.3. Ground level DC ion current density

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